

Solutions to systems of linear equations with backward substitution

Note, this does not include applications, as for now, it is just necessary to learn how to find these solutions. For each question, let the columns be associated with the variables $x_1, x_2, \dots$. Recommended: work with your peers, and each do some of each set, and explain your results to your peers.

1. For each of the following, determine if there are no solutions, one solution, or infinitely many solutions? For the latter two, find all solutions.

$$(4 \mid 2), (0 \mid 2), (3 \mid 0), (0 \mid 0)$$

2. For each of the following, determine if there are no solutions, one solution, or infinitely many solutions? For the latter two, find all solutions.

$$(4 \ 2 \mid 6), (0 \ 2 \mid 6), (0 \ 2 \mid 6), (0 \ 0 \mid 4), (0 \ 0 \mid 0)$$

Note: you may be wondering where systems of linear equations like $(0 \ 0 \mid 0)$ come from; however, in applications (such as finding eigenvalues), it may turn out that you get a bizarre equation or system of equations as a natural consequence of the conditions.

3. For each of the following, determine if there are no solutions, one solution, or infinitely many solutions? For the latter two, find all solutions.

$$\begin{pmatrix} 4 & 2 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}, \begin{pmatrix} 0 & 4 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 4 \\ 0 & 1 \end{pmatrix}$$

4. For each of the following, determine if there are no solutions, one solution, or infinitely many solutions? For the latter two, find all solutions.

$$\begin{pmatrix} 4 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 4 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 4 & 2 & 2 \\ 0 & 3 & 6 \end{pmatrix}, \begin{pmatrix} 4 & 2 & 2 \\ 0 & 3 & 6 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 4 & 2 & 2 \\ 0 & 3 & 6 \\ 0 & 0 & 1 \end{pmatrix}$$

5. For each of the following, determine if there are no solutions, one solution, or infinitely many solutions? For the latter two, find all solutions.

$$\begin{pmatrix} 2 & 1 & -2 & 5 \\ 0 & 3 & -1 & 5 \\ 0 & 0 & 2 & -4 \end{pmatrix}, \begin{pmatrix} 2 & 1 & -2 & 5 \\ 0 & 3 & -1 & 5 \\ 0 & 0 & 0 & -4 \end{pmatrix}, \begin{pmatrix} -5 & 1 & -2 & 5 \\ 0 & 2 & -1 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -5 & 1 & -2 & 5 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -5 & 1 & -2 & 5 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} -5 & 1 & -2 & 5 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -5 & 1 & -2 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 1 & -2 & 5 \\ 0 & 3 & -1 & 5 \\ 0 & 0 & 2 & -4 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 1 & -2 & 5 \\ 0 & 3 & -1 & 5 \\ 0 & 0 & 2 & -4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

6. For each of the following, determine if there are no solutions, one solution, or infinitely many solutions? For the latter two, find all solutions.

$$\begin{pmatrix} 2 & 1 & -1 & 2 & 4 \\ 0 & -5 & 3 & -1 & -2 \\ 0 & 0 & -1 & 1 & 4 \\ 0 & 0 & 0 & -3 & 6 \end{pmatrix}, \begin{pmatrix} 2 & 1 & -1 & 2 & 4 \\ 0 & -5 & 3 & -1 & -2 \\ 0 & 0 & -1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 1 & -1 & 2 & 4 \\ 0 & -5 & 3 & -1 & -2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 1 & -1 & 2 & 4 \\ 0 & 0 & 4 & -1 & -2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 2 & 1 & -1 & 2 & 4 \\ 0 & 0 & 4 & -1 & -2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 1 & -1 & 2 & 4 \\ 0 & 0 & 4 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 1 & -1 & 2 & 4 \\ 0 & -5 & 4 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 1 & -1 & 2 & 4 \\ 0 & -5 & 4 & -1 & -2 \\ 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 2 & 1 & -2 & 5 \\ 0 & 0 & 3 & -1 & 5 \\ 0 & 0 & 0 & 2 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 & 1 & -2 & 5 \\ 0 & 0 & 3 & -1 & 5 \\ 0 & 0 & 0 & 2 & -4 \\ 0 & 0 & 0 & 0 & 6 \end{pmatrix}$$

7. The only question for this assignment is: Have you tried enough of these to understand backward substitution if the augmented matrix is in row-echelon form?